Coastal Hydroscience Analysis, Modeling, & Predictive Simulations Laboratory

CHAMPS Lab
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Commonality of Hydrologic Models across Time Scales

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Water Balance across Time Scales

- Mass conservation equation
- Storage change = Inflow – Outflow
- Applicable to any time scale

Water Balance across Time Scales

- Event scale: $E \approx 0$
  - Storage change = Rainfall - Runoff
- Mean annual: $\Delta S \approx 0$
  - Precipitation = Runoff + Evaporation
- Controlling factors vary with time scales

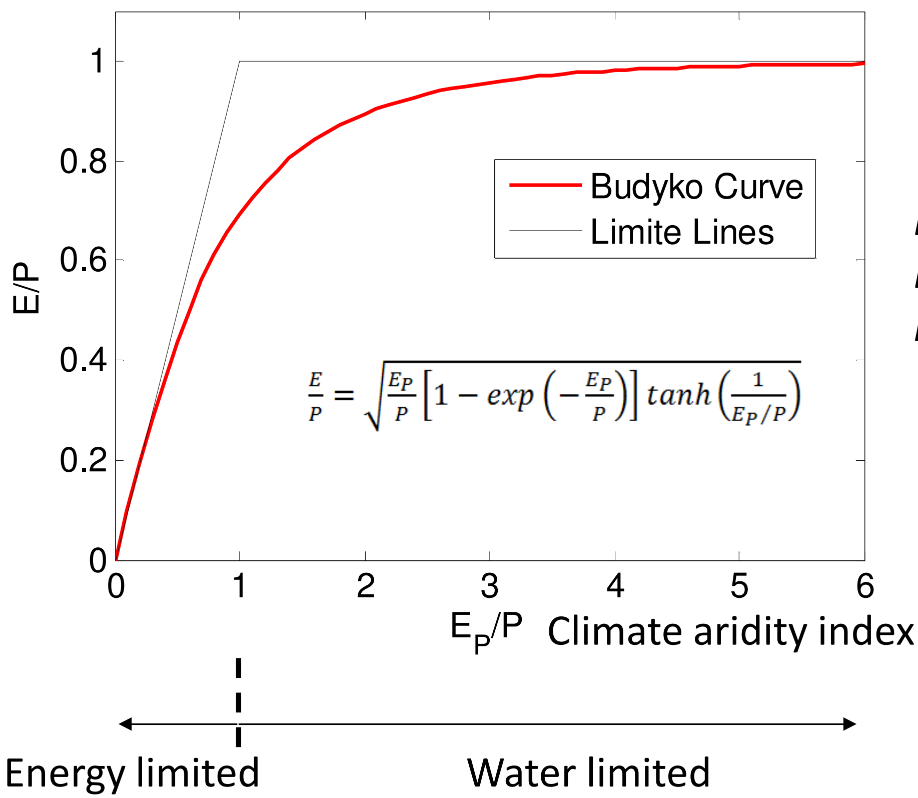
Existing Hydrologic Models

- Budyko model
  - Mean annual water balance

- "abcd" model
  - Monthly or daily hydrologic model

- SCS curve number model
  - Direct runoff at the event scale

Budyko Model

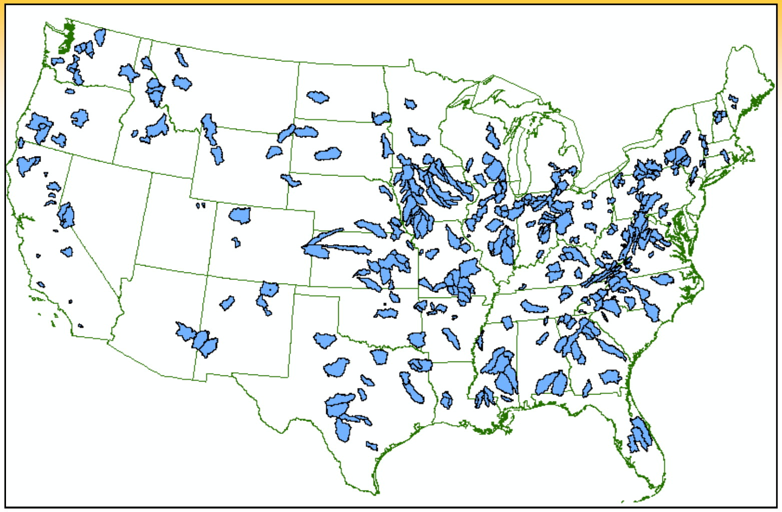


E: evaporation

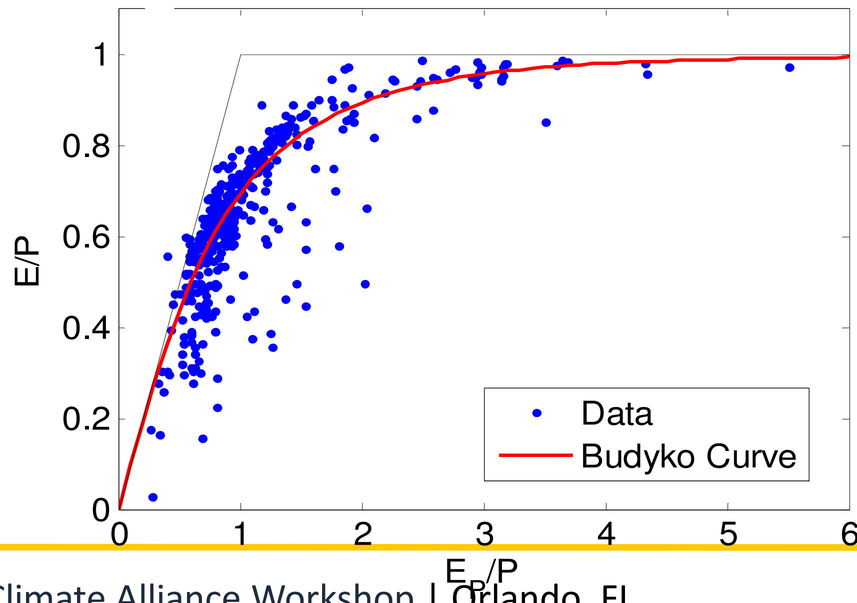
E_p: potential evaporation

P: precipitation

Budyko Curve



MOPEX Watersheds



"abcd" Model

- Water balance $S_t - S_{t-1} = P_t - E_t - Q_t$

$$\Downarrow$$

$$(P_t + S_{t-1}) = (S_t + E_t) + Q_t$$

Available water $\Downarrow$ $\qquad$ $\Downarrow$ Evaporation opportunity

$$W_t = Y_t + Q_t$$

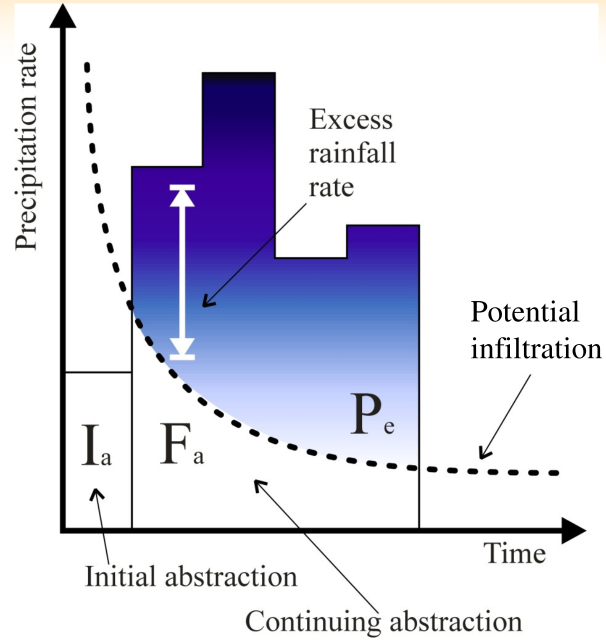
- Partitioning $Y_t = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}}$

SCS Model

- Rainfall excess or direct runoff

$$P_e = \frac{(P - I_a)^2}{P - I_a + S}$$

S: Potential of retention



- Partitioning

	Actual	Potential
$P - I_a$ →	F_a	S
	P_e	$P - I_a$ when $F_a = 0$

Proportionality

$$\frac{F_a}{S} = \frac{P_e}{P - I_a}$$

Generalized Proportionality Hypothesis

- During any time interval

$$Z \to X_0, \quad Z \to Z - X_0$$

$$Z - X_0 \to X - X_0, \quad Z - X_0 \to Y$$

Potential

$X_P - X_0$

$Z - X_0$

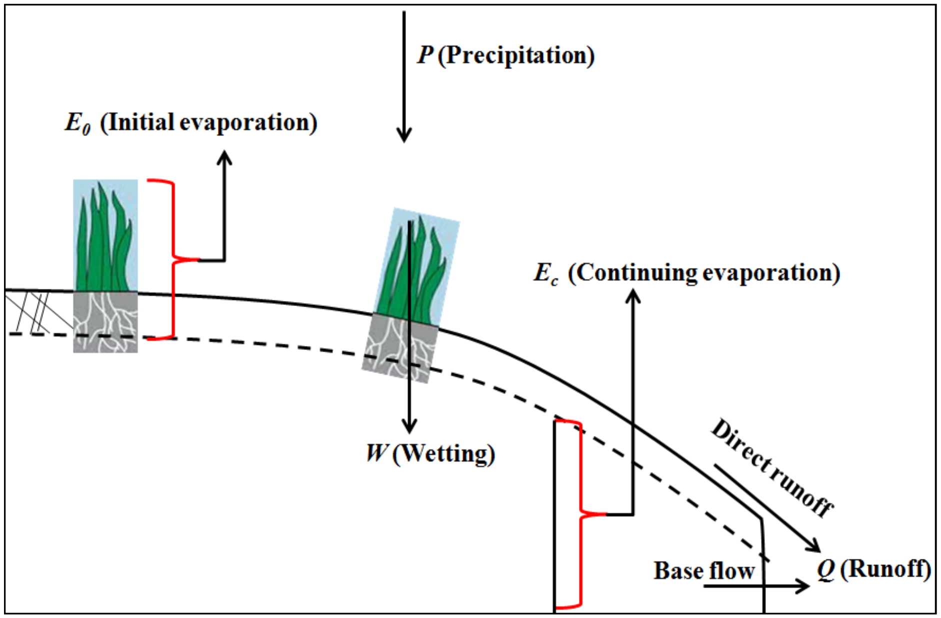
- Proportionality $\dfrac{X - X_0}{X_P - X_0} = \dfrac{Y}{Z - X_0}$

Hypothesis

- The generalized proportionality principle is the temporal scaling pattern of water balance

    - Applied to mean annual water balance

Proportionality for Mean Annual Water Balance



Proportionality

$$\frac{E_c}{E_p - E_0} = \frac{Q}{P - E_0}$$

$$P \begin{cases} E_0 \\ P - E_0 \begin{cases} E_c \\ Q \end{cases} \end{cases}$$

Potential evaporation

Potential

$$E_p - E_0$$

$$P - E_0$$

Derived Budyko Equation

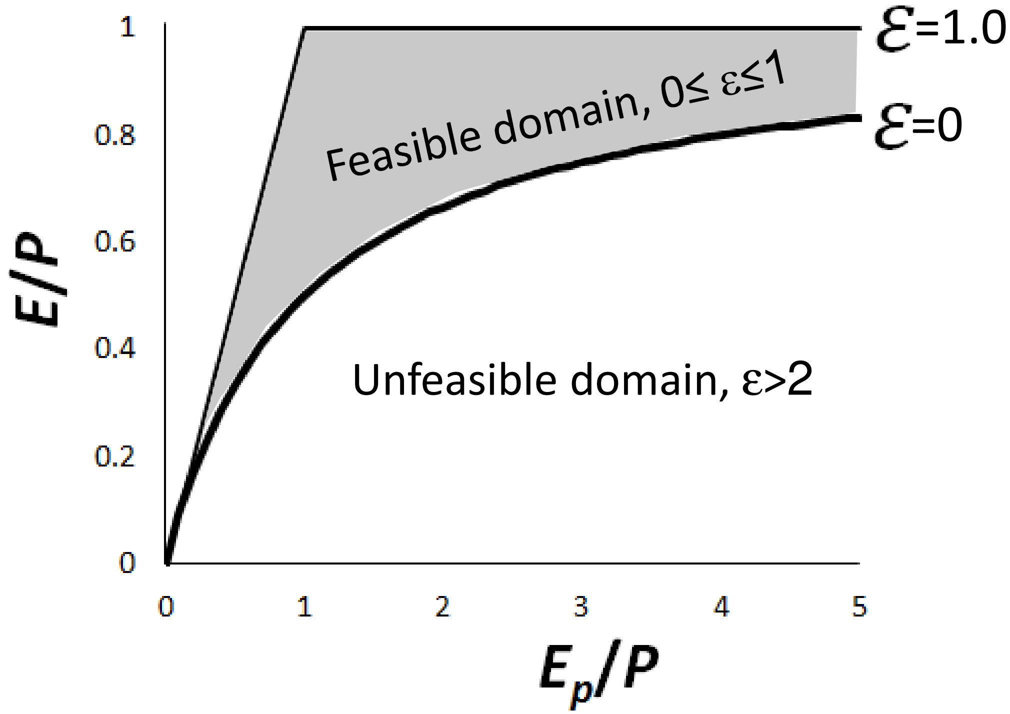
$$\frac{E}{P} = \frac{1+E_p/P-\sqrt{\left(1+E_p/P\right)^2-4\varepsilon(2-\varepsilon)E_p/P}}{2\varepsilon(2-\varepsilon)}$$

$$\varepsilon = \lambda/H$$

ε can also be interpreted as $\varepsilon = E_0/E$

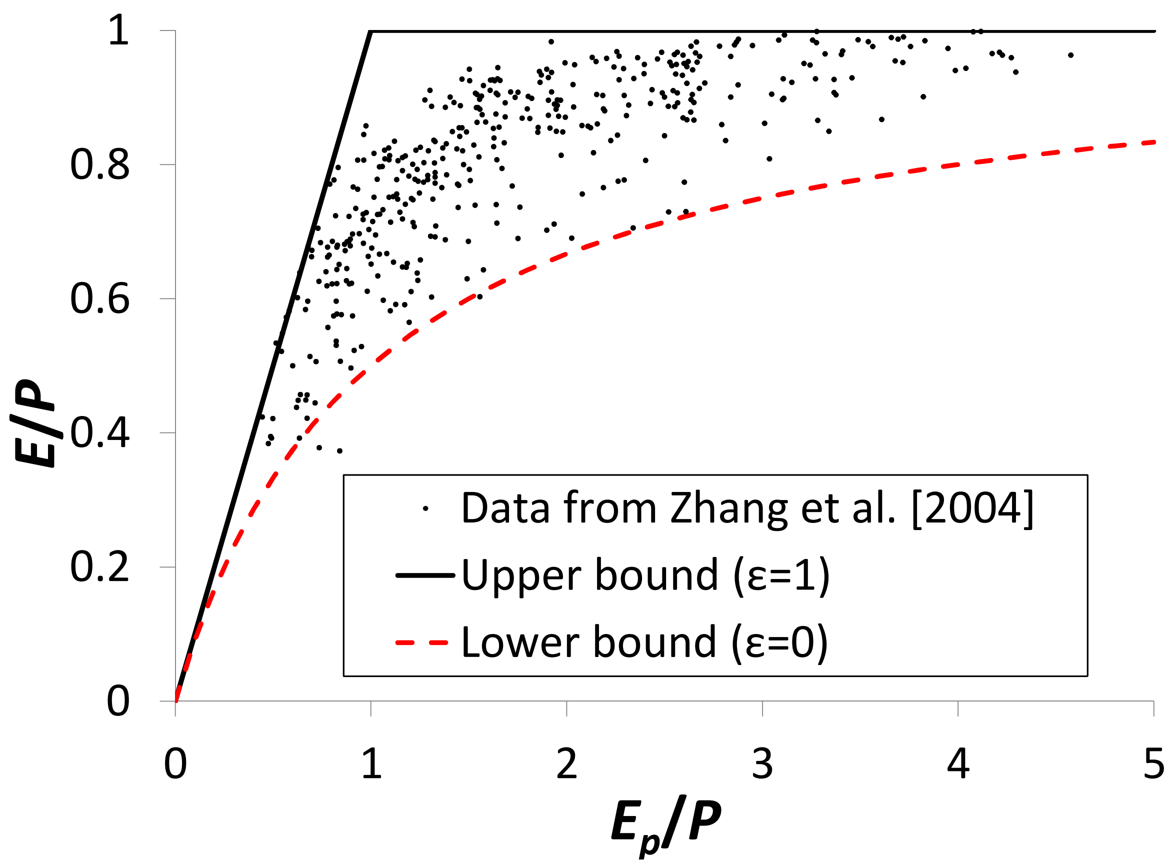
$$0 \leq \varepsilon \leq 1$$

Feasible Space of Watersheds



Climate aridity index

Watersheds from the World



“abcd” and Derived Budyko Models

- Budyko model

$$\frac{E}{P} = \frac{1+E_p/P-\sqrt{\left(1+E_p/P\right)^2-4\varepsilon(2-\varepsilon)E_p/P}}{2\varepsilon(2-\varepsilon)}$$

Same functional form!

- “abcd” model

$$Y_t = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}}$$

Commonality of Hydrologic Models across Time Scales

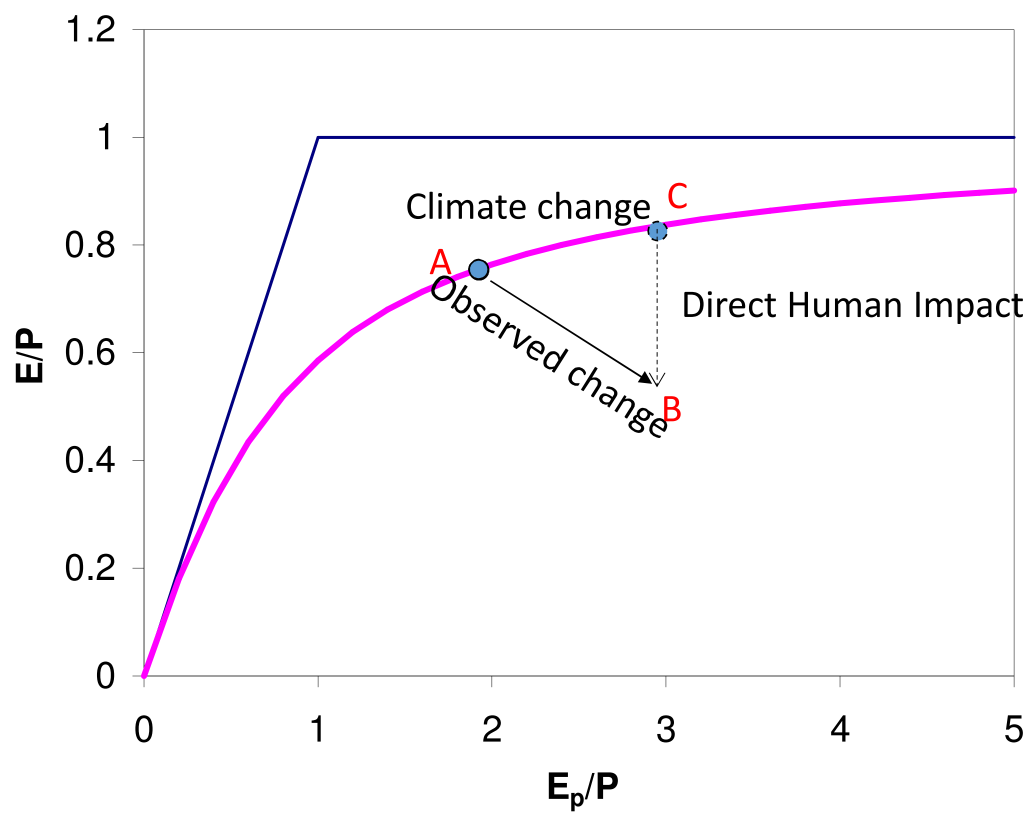
- Proportionality

  Mean annual: Budyko

  Monthly/daily: “abcd”

  Event: SCS

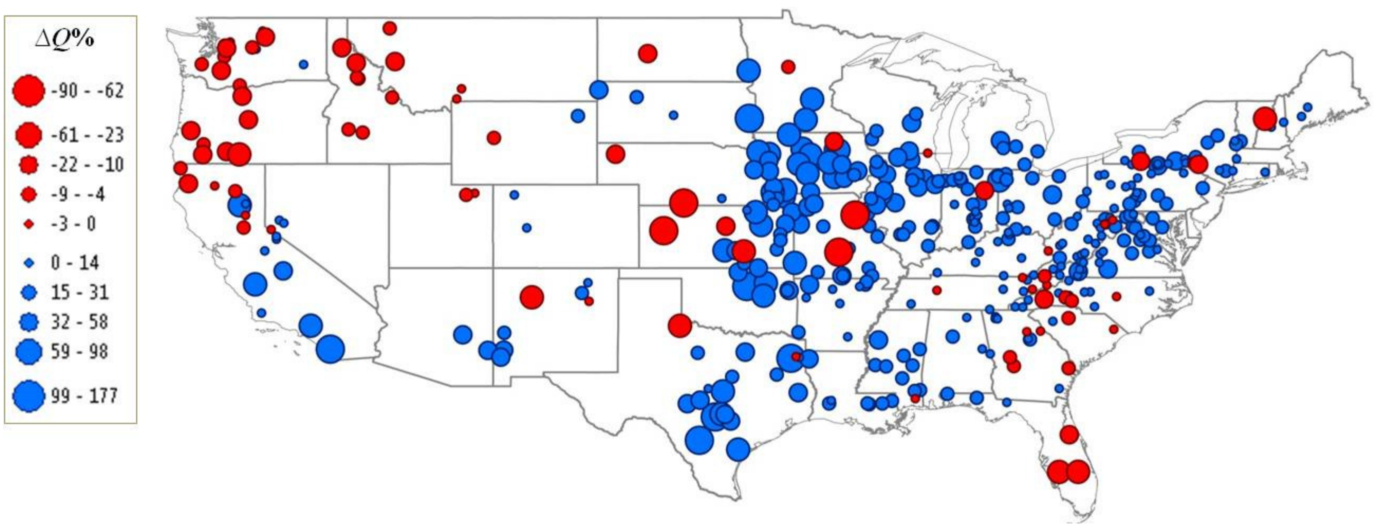
Decomposition of Climate and Human Impacts



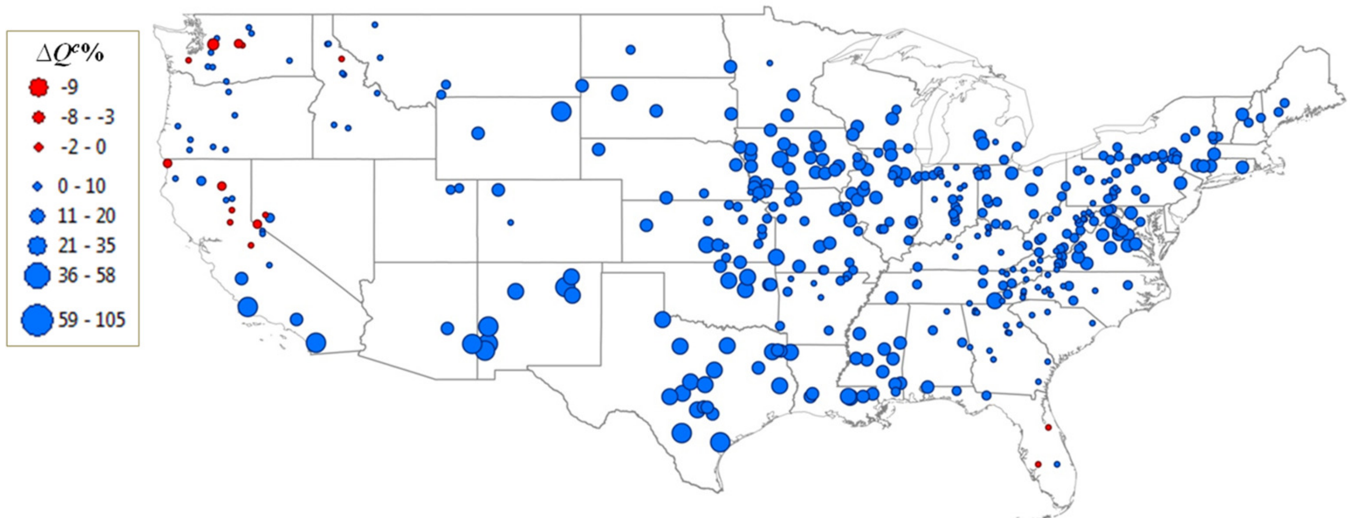
Application to MOPEX Watersheds

- Pre-change: 1948-1970
- Post-change: 1971-2003
- Derived mean annual water balance equation

Observed Streamflow Change

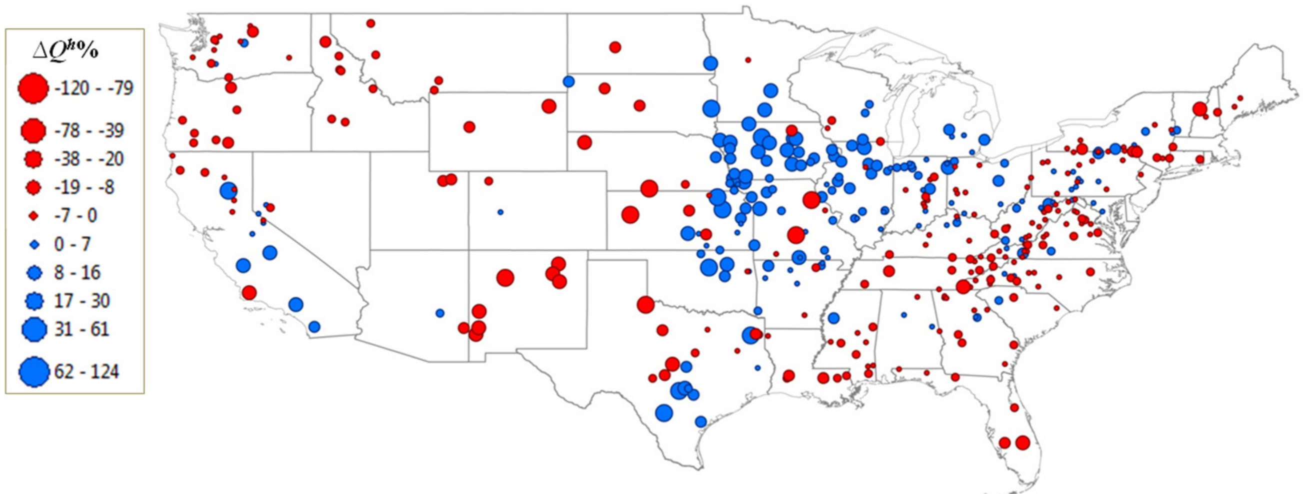


Climate Induced Streamflow Change



Increased precipitation

Direct Human Induced Streamflow Change



Regional patterns

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References

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Thank you!

Continued…

- Define $\lambda = E_0/W$

- Horton index

$$H = E/W$$

$$\Downarrow$$

$$\frac{W}{P} = \frac{W \cdot E}{P \cdot E} = \frac{E}{P} \cdot \frac{1}{H}$$

$$\frac{E_c}{E_p - E_0} = \frac{Q}{P - E_0}$$

$$\Downarrow$$

$$\frac{E - E_0}{E_p - E_0} = \frac{P - E}{P - E_0}$$

$$\Downarrow$$

$$\frac{E - \lambda W}{E_p - \lambda W} = \frac{P - E}{P - \lambda W}$$

$$\Downarrow$$

$$\frac{E/P - \lambda W/P}{E_p/P - \lambda W/P} = \frac{1 - E/P}{1 - \lambda W/P}$$

$$\Downarrow$$

$$\frac{E/P - \frac{\lambda}{H} E/P}{E_p/P - \frac{\lambda}{H} E/P} = \frac{1 - E/P}{1 - \frac{\lambda}{H} E/P}$$

Meaning of Lower Bound

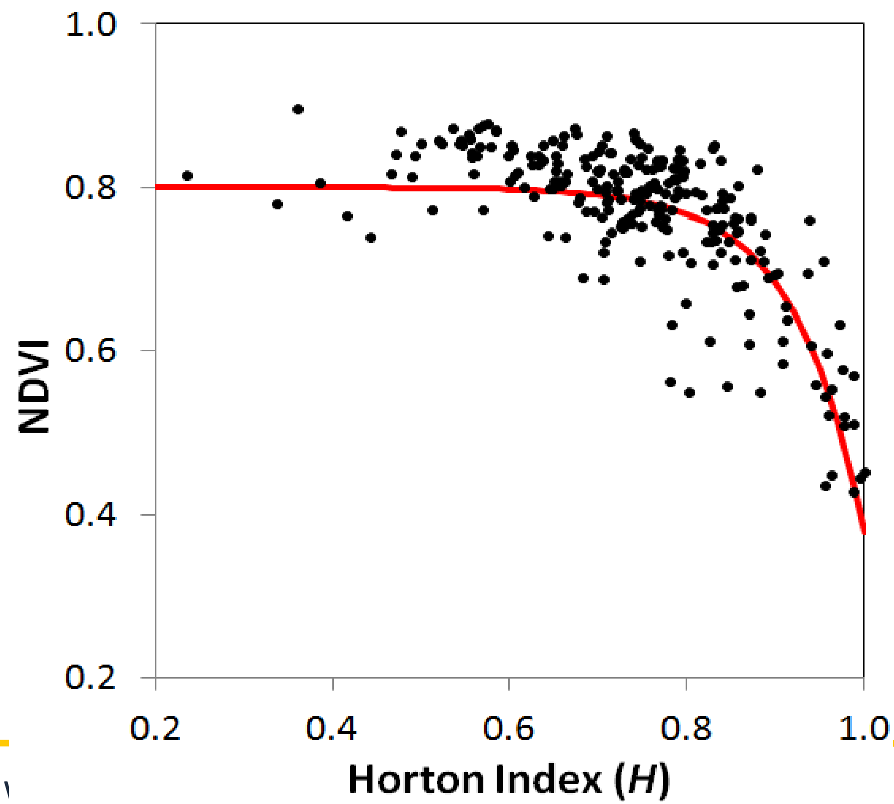
- Lower bound $\lim_{\varepsilon \to 0} \frac{E}{P} = \left[1 + \left(\frac{E_p}{P}\right)^{-1}\right]^{-1}$

		Potential
Corresponding to P	$\nearrow E$	E_P
	$\searrow Q$	P

$$\frac{E}{E_P} = \frac{Q}{P}$$

- Lower bound means $\frac{E}{E_p} \geq \frac{Q}{P}$

Physical Controls on the Parameter



Physical Controls on the Parameter