


Coastal Hydroscience Analysis, Modeling, & Predictive Simulations Laboratory

CHAMPS Lab
http://champs.cecs.ucf.edu

# Commonality of Hydrologic Models across Time Scales

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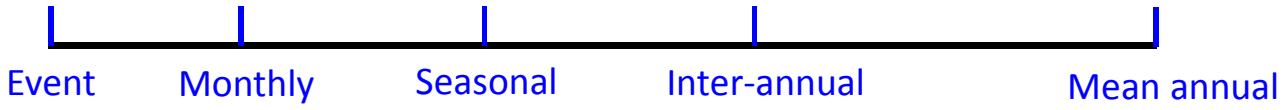
# Water Balance across Time Scales

- Mass conservation equation
- Storage change = Inflow – Outflow
- Applicable to any time scale

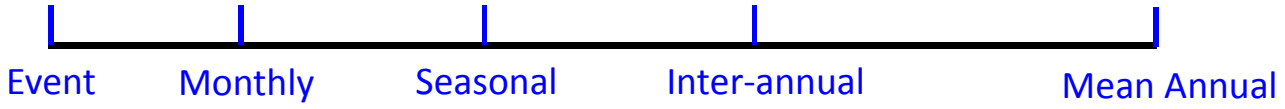


# Water Balance across Time Scales

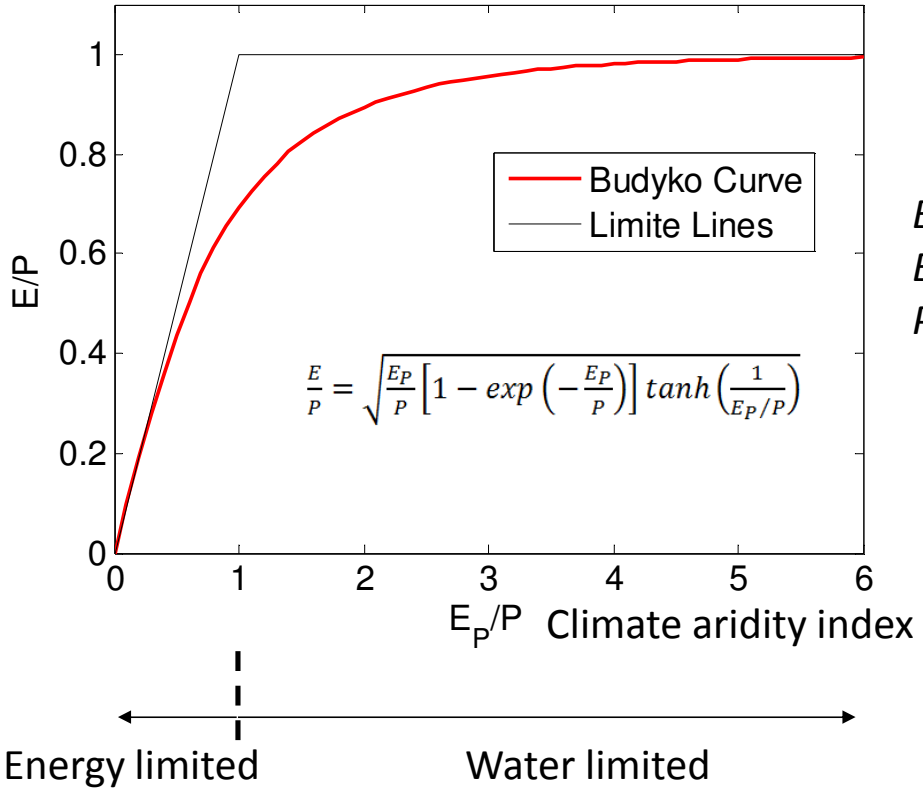
- Event scale: $E \approx 0$
  - Storage change = Rainfall - Runoff
- Mean annual: $\Delta S \approx 0$
  - Precipitation = Runoff + Evaporation
- Controlling factors vary with time scales

# Existing Hydrologic Models

- Budyko model
  - Mean annual water balance
- "abcd" model
  - Monthly or daily hydrologic model
- SCS curve number model
  - Direct runoff at the event scale

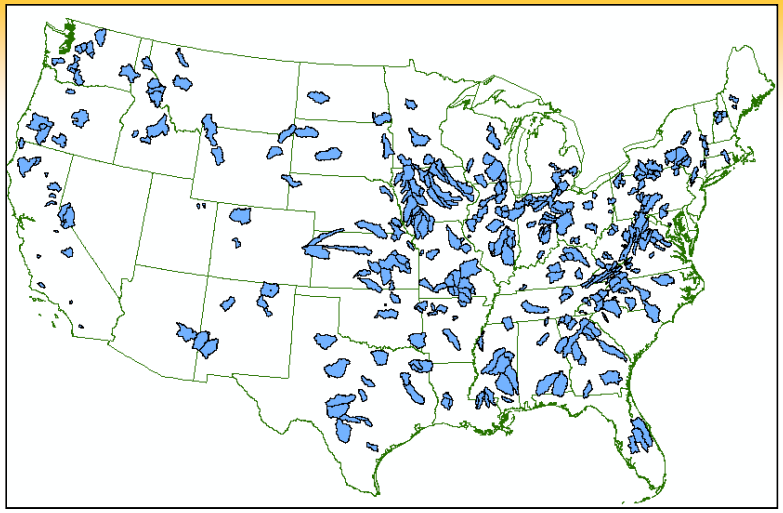


# Budyko Model

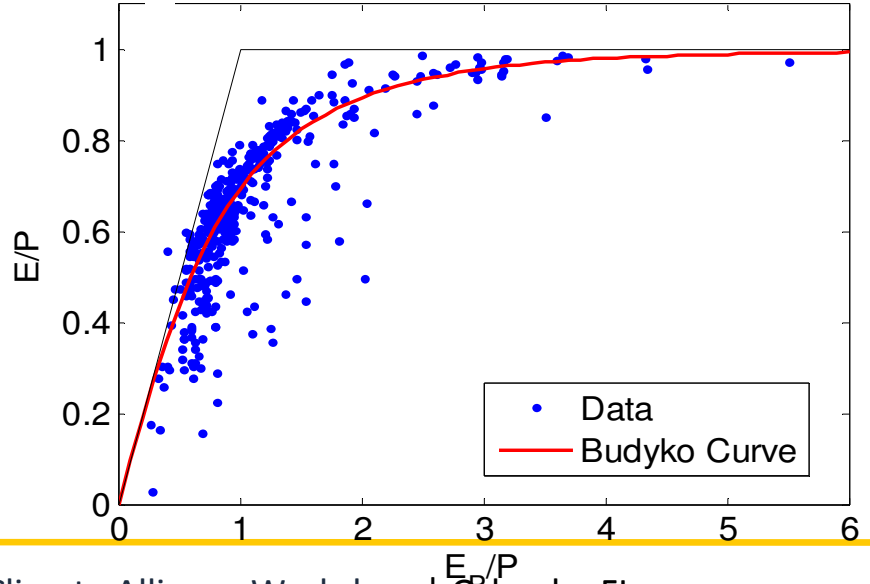



*E*: evaporation
$E_p$: potential evaporation
*P:* precipitation

# Budyko Curve



MOPEX Watersheds



# "abcd" Model

- Water balance $S_t - S_{t-1} = P_t - E_t - Q_t$

$$(P_t + S_{t-1}) = (S_t + E_t) + Q_t$$

Available water $W_t$ = Evaporation opportunity $Y_t$ + $Q_t$

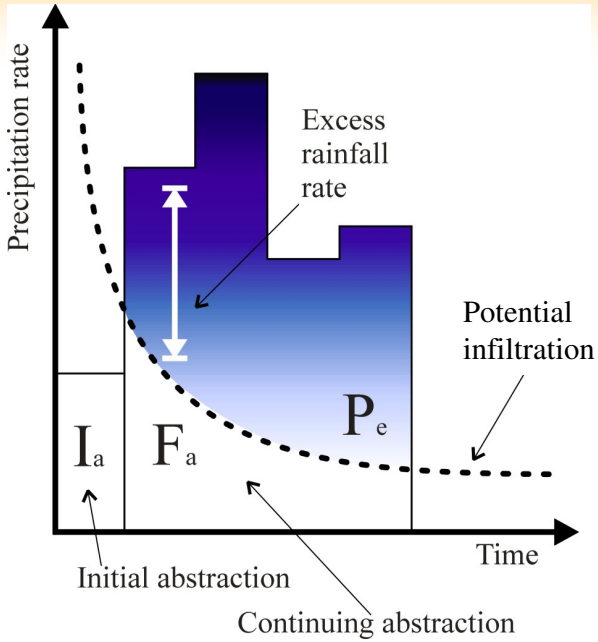
- Partitioning $$Y_t = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}}$$

# SCS Model

- Rainfall excess or direct runoff

$$P_e = \frac{(P - I_a)^2}{P - I_a + S}$$

*S*: Potential of retention




- Partitioning

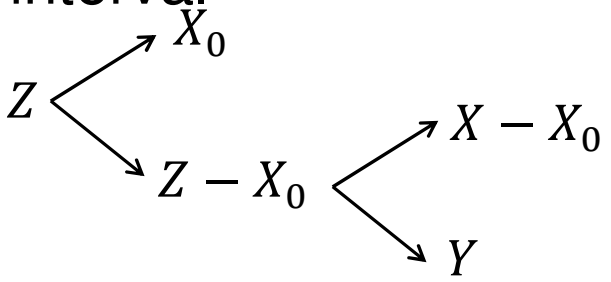
| | Actual | Potential |
|---|---|---|
| $P - I_a$ | $F_a$ | $S$ |
| | $P_e$ | $P - I_a$ when $F_a = 0$ |

Proportionality

$$\frac{F_a}{S} = \frac{P_e}{P - I_a}$$

# Generalized Proportionality Hypothesis

- During any time interval




Potential

$X_P - X_0$

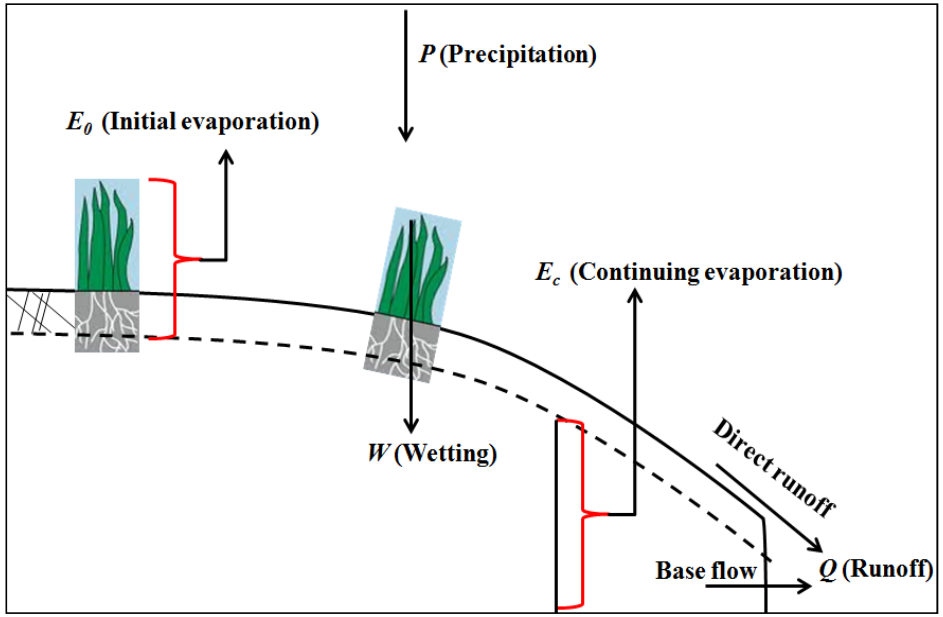
$Z - X_0$

- Proportionality $\dfrac{X - X_0}{X_P - X_0} = \dfrac{Y}{Z - X_0}$

# Hypothesis

- The generalized proportionality principle is the temporal scaling pattern of water balance
  - Applied to mean annual water balance

# Proportionality for Mean Annual Water Balance




Proportionality

$$\frac{E_c}{E_p - E_0} = \frac{Q}{P - E_0}$$

Potential evaporation

Potential

$$P \begin{cases} E_0 \\ P - E_0 \end{cases} \quad P - E_0 \begin{cases} E_c \\ Q \end{cases} \quad E_p - E_0 \quad P - E_0$$

# Derived Budyko Equation

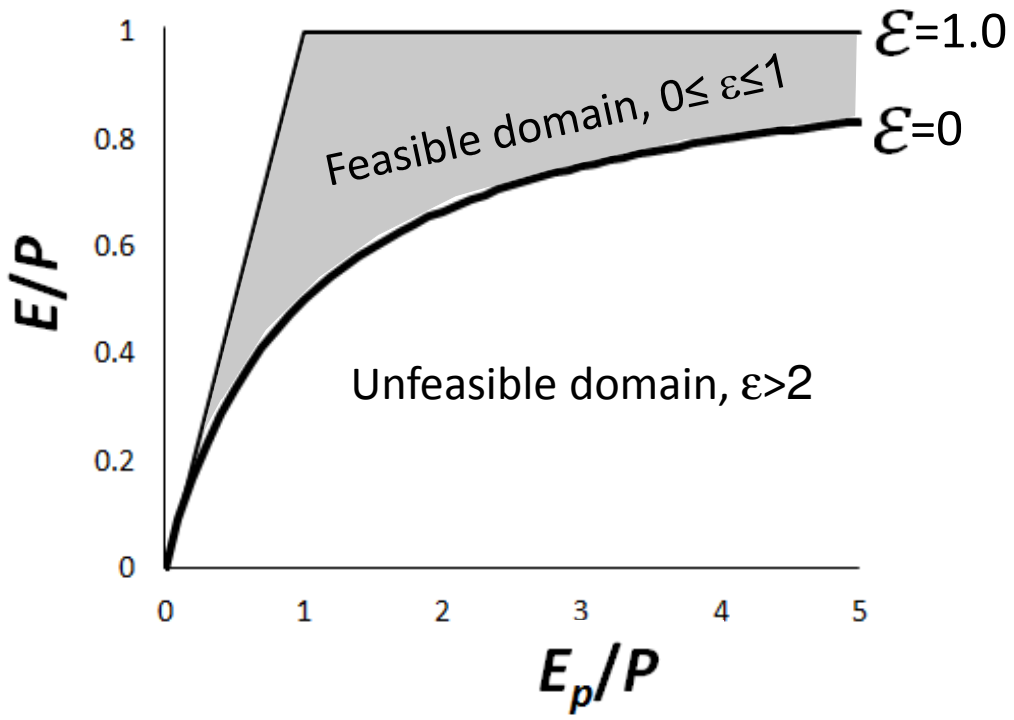
$$\frac{E}{P} = \frac{1+E_p/P-\sqrt{\left(1+E_p/P\right)^2-4\varepsilon(2-\varepsilon)E_p/P}}{2\varepsilon(2-\varepsilon)}$$

$$\varepsilon = \lambda/H$$

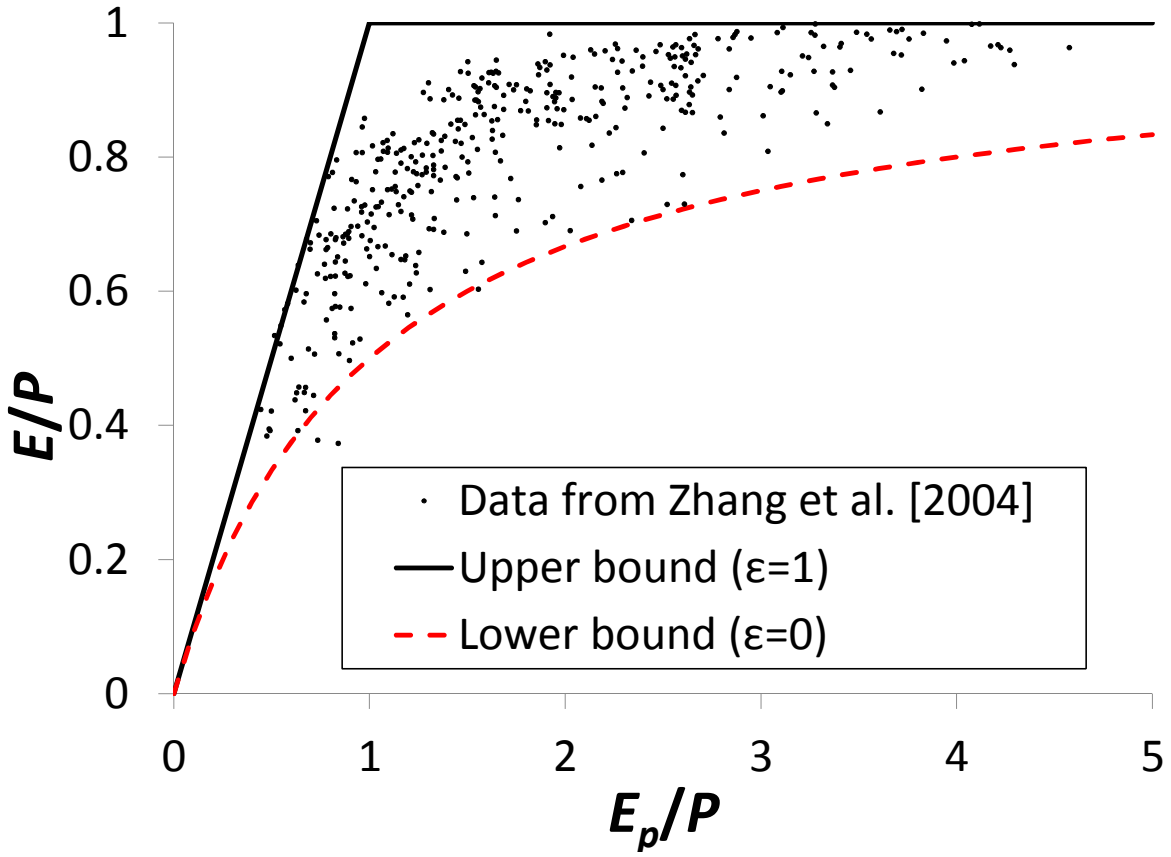
$\varepsilon$ can also be interpreted as $\varepsilon = E_0/E$

$$0 \leq \varepsilon \leq 1$$

# Feasible Space of Watersheds



# Watersheds from the World



# "abcd" and Derived Budyko Models

- Budyko model

$$\frac{E}{P} = \frac{1+E_p/P-\sqrt{\left(1+E_p/P\right)^2-4\varepsilon(2-\varepsilon)E_p/P}}{2\varepsilon(2-\varepsilon)}$$

Same functional form!

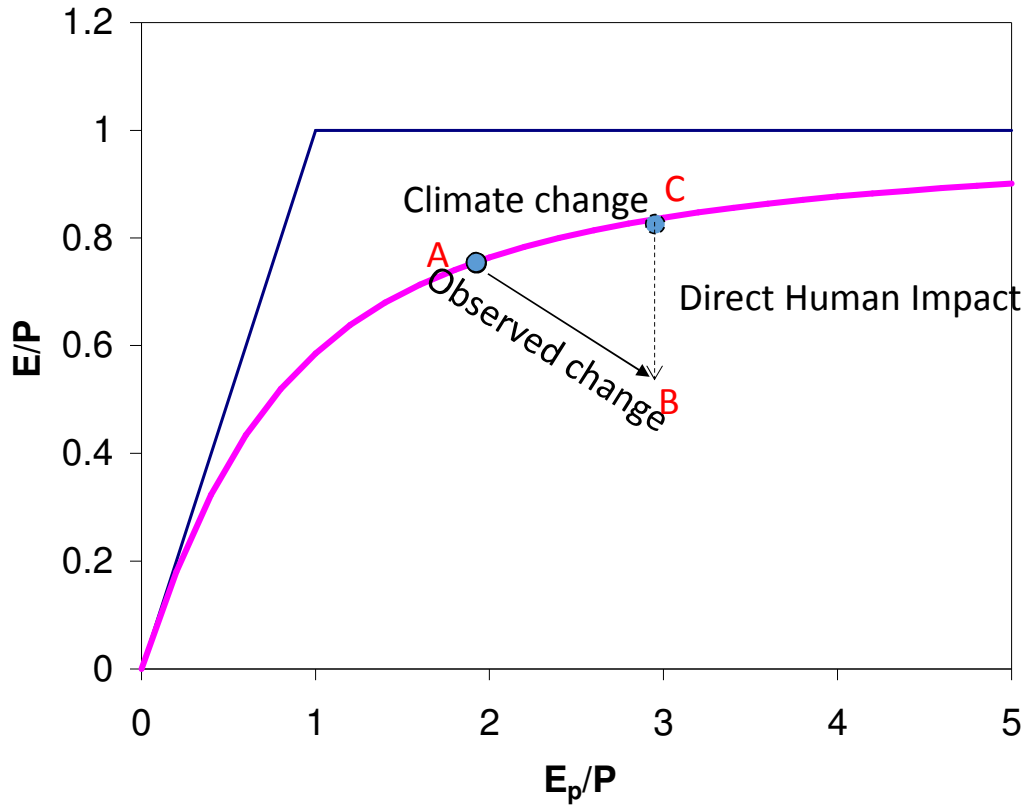
- "abcd" model

$$Y_t = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}}$$

# Commonality of Hydrologic Models across Time Scales

- Proportionality
  - Mean annual: Budyko
  - Monthly/daily: "abcd"
  - Event: SCS

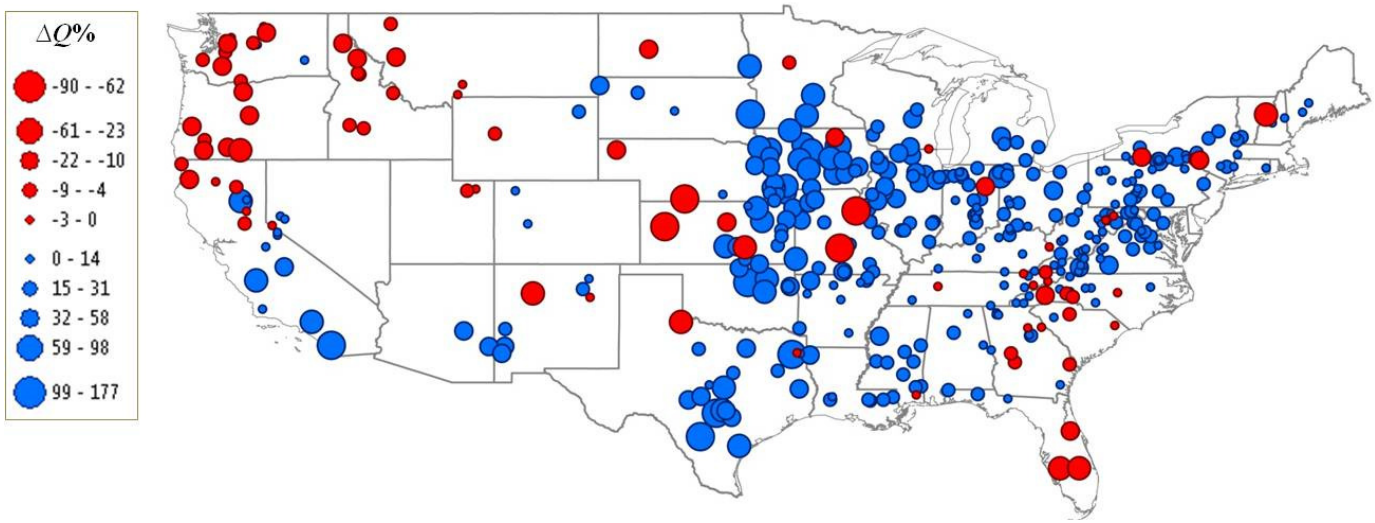
# Decomposition of Climate and Human Impacts



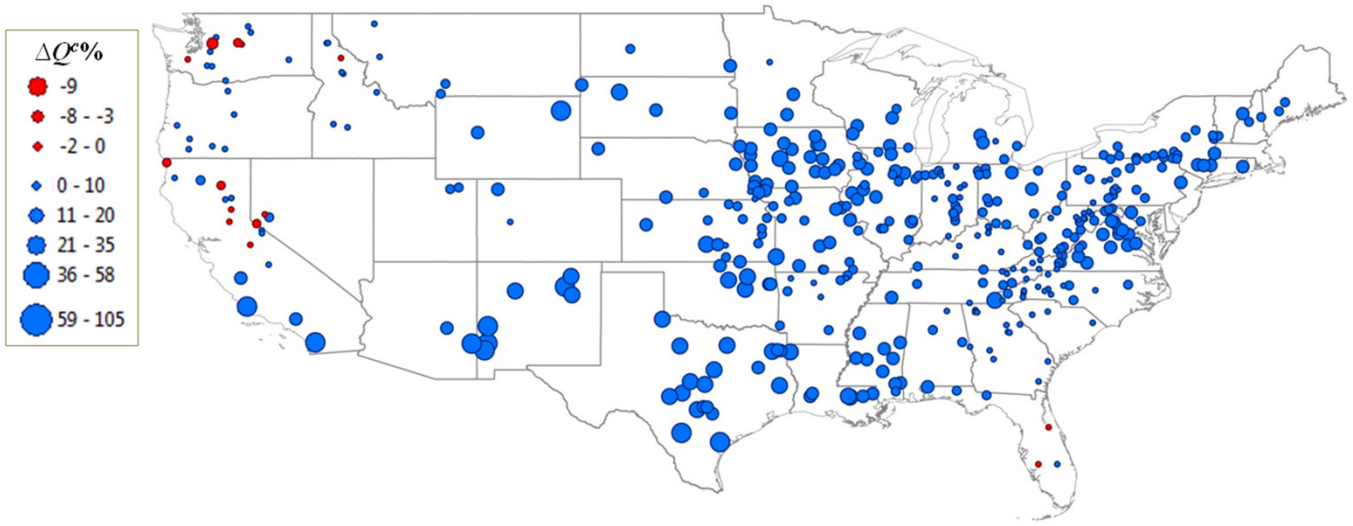
# Application to MOPEX Watersheds

- Pre-change: 1948-1970
- Post-change: 1971-2003
- Derived mean annual water balance equation

# Observed Streamflow Change

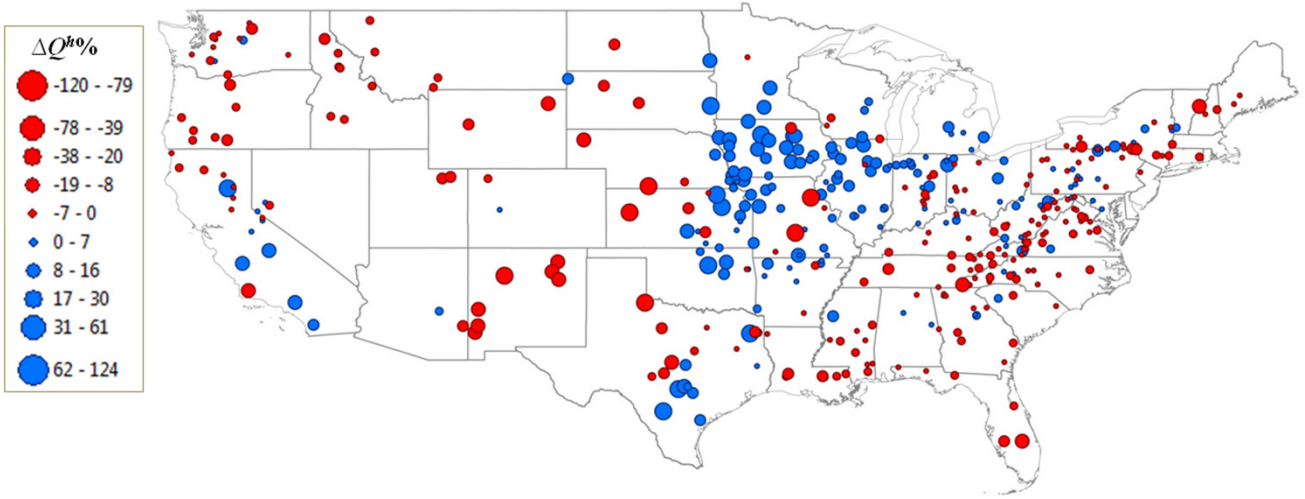


# Climate Induced Streamflow Change




Increased precipitation

# Direct Human Induced Streamflow Change




Regional patterns

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CHAMPS Lab
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- Funding sources

# References


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- Wang, D. and M. Hejazi (2011), Quantifying the relative contribution of the climate and direct human impacts on mean annual streamflow in the contiguous United States, *Water Resources Research*, 47.

# Thank you!

# Continued…

- Define $\lambda = E_0/W$

- Horton index

$$H = E/W$$

$$\Downarrow$$

$$\frac{W}{P} = \frac{W \cdot E}{P \cdot E} = \frac{E}{P} \cdot \frac{1}{H}$$

$$\frac{E_c}{E_p - E_0} = \frac{Q}{P - E_0}$$

$$\Downarrow$$

$$\frac{E - E_0}{E_p - E_0} = \frac{P - E}{P - E_0}$$

$$\Downarrow$$

$$\frac{E - \lambda W}{E_p - \lambda W} = \frac{P - E}{P - \lambda W}$$

$$\Downarrow$$

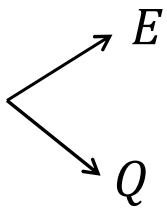
$$\frac{E/P - \lambda W/P}{E_p/P - \lambda W/P} = \frac{1 - E/P}{1 - \lambda W/P}$$

$$\Downarrow$$

$$\frac{E/P - \frac{\lambda}{H} E/P}{E_p/P - \frac{\lambda}{H} E/P} = \frac{1 - E/P}{1 - \frac{\lambda}{H} E/P}$$

# Meaning of Lower Bound

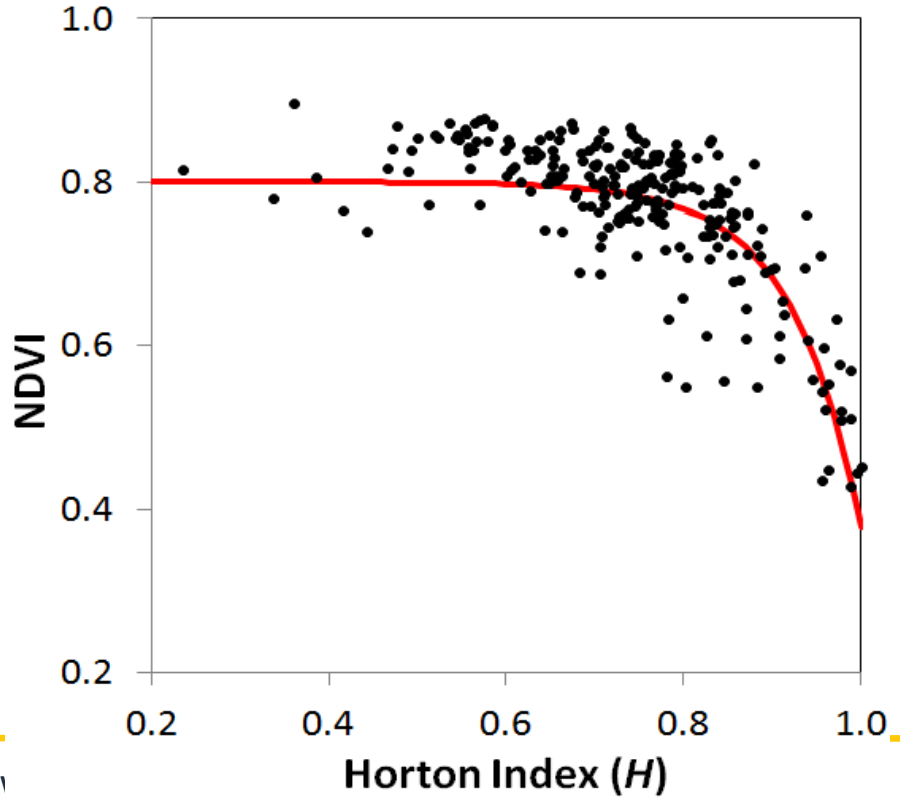
- Lower bound $\lim_{\varepsilon \to 0} \frac{E}{P} = \left[1 + \left(\frac{E_p}{P}\right)^{-1}\right]^{-1}$

- Corresponding to $P$ 

Potential
$E_P$
$P$
$\frac{E}{E_P} = \frac{Q}{P}$

- Lower bound means $\frac{E}{E_p} \geq \frac{Q}{P}$

# Physical Controls on the Parameter



# Physical Controls on the Parameter